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IR-renormalon contributions to the structure functions g_3 and g_5

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Abstract. We calculate the leading $1/N_f$ perturbative contributions to the polarized nonsinglet structure functions g_3 and g_5 to all orders in α_s. The contributions from the first renormalon pole are determined. It is a measure for the ambiguity of the perturbative calculation and is hypothetically assumed to dominate the power corrections. The corrections Δg_3 and Δg_5 are given as functions of the Bjorken variable x and turn out to be negligible. The anomalous dimensions of the leading twist operators are obtained in the next-to-leading order.

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It is well known that the perturbation series for moments of twist-2 structure functions is an asymptotic one. This property can be studied in detail in the $1/N_f$-limit, in which the complete series can be calculated explicitly. Formally this series is given by an integral over the positive real axis in the Borel plane. This integral is ambiguous due to singularities on the integration path, the so called IR-renormalon poles. The residues of these poles are a measure for the ambiguity of the perturbation series. The so-called hypothesis of UV-dominance allows furthermore to interpret this ambiguity as an estimate for the power corrections. This interpretation is controversial but it clearly provides one piece of information to clarify the relationship between perturbative $1/\ln Q^2$ corrections and the $1/Q^2$ power corrections. The program just sketched was already applied to all twist-2 structure functions except $g_3(x,Q^2)$ and $g_5(x,Q^2)$ [1]–[3]. In this contribution we investigate these remaining two cases. Let us note that similar renormalon analyses have recently been applied to a large range of other QCD observables [4]–[12].

The Borel transformation of a perturbative series

$$R = r_0 a + r_1 a^2 + r_2 a^3 = \ldots = \sum_{n=0}^{\infty} r_n a^{n+1} , \quad a = \alpha_s \cdot 4\pi \tag{1}$$

is defined as

$$B[R](u) = \sum_{n=0}^{\infty} u^n \frac{r_n}{n!} . \tag{2}$$

R can be reobtained from its Borel transform by an integration over the positive real axis as

$$R = \int_0^{\infty} \mathrm{d}u \, \mathrm{e}^{-u/a} B[R](u) . \tag{3}$$

The coefficients of the original power series can also be obtained individually by taking the derivatives with respect to u

$$r_k = \left. \frac{\mathrm{d}^k}{\mathrm{d}u^k} B[R](u) \right|_{u=0} . \tag{4}$$

$B[R]$ has pole singularities on the real axis, the so-called renormalons [13]. The poles on the positive u-axis, which are called IR-renormalons because they can be traced back to low momentum contribution to the loop integrals, lead to ambiguities in the back-transformation (3) because it is unclear whether they have to be passed above or below the real axis. The fact that no unambiguous back-transformation exists reflects the fact that the perturbative expansions are asymptotic [14,15]. The ambiguities are of the order of magnitude

$$\Delta R = \mathrm{e}^{-u/a} \mathrm{Res}\Big(B[R](u) \Big) \Big|_{u=\text{pole position}} , \tag{5}$$

and can be interpreted as a measure for generic uncertainties of perturbative predictions or in other words as an estimate for corrections beyond leading twist perturbation theory [4].

In connection with the investigation of renormalons the NNA-approximation (naive non-Abelianization) [16] is of particular interest because in the Borel plane it leads to an effective gluon propagator of a very simple form allowing a calculation to all orders in the coupling constant. In the NNA-approximation we start with a restriction to the leading $1/N_f$-terms (N_f: number of quark flavors), which is the sum of all diagrams with only one exchanged gluon but an arbitrary number of quark loops.

The missing terms are then approximated by the replacement $N_f \to -24\pi^2\beta_0 = N_f - 33/2$, which corresponds to a restriction to the leading terms of an expansion in powers of the one loop QCD β-function. The resummation of all corresponding diagrams leads to the Borel transformed effective gluon propagator

$$B[g^2 D^{ab}_{\mu\nu}(k)](s) = \delta^{ab}\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{k^2}\left(\frac{\mu^2 \mathrm{e}^{-c}}{-k^2}\right)^s \tag{6}$$

with the new variable $s := \beta_0 u$ [5,6,17]. c is a renormalization scheme dependent constant, in the $\overline{\mathrm{MS}}$-scheme $c = -5/3$. The expression (6) differs from the original gluon propagator essentially only by the power of k^2 in the denominator. Consequently a calculation of a Borel transform in the NNA-approximation in all orders of the coupling constant is not more complicated than the corresponding normal next-to-leading-order calculation.

We now apply the described method to the structure functions g_3 and g_5 measurable in polarized deep inelastic lepton-nucleon scattering. These structure functions are defined by the following terms in the decomposition of the hadronic scattering tensor

$$W_{\mu\nu} = -\frac{m_N(p_\mu S_\nu + S_\mu p_\nu)}{p\cdot q} g_3 + \frac{2\,m_N S\cdot q\, g_{\mu\nu}}{p\cdot q} g_5 + \dots\,. \tag{7}$$

We adopted the conventions of [18], a comparison with other definitions used in the literature is given in [19]. Since the contributions to $W_{\mu\nu}$ shown in (7) are parity violating they involve weak interactions. We are looking at the case of pure Z-boson exchange and the interference part of Z- and γ-exchange. In order to avoid operator mixing we consider the nonsinglet part, which is obtained by taking the difference between proton- and neutron-structure functions [20,21]. To simplify the notation we write $g_j := g^p_j - g^n_j$, $j = 3, 5$. Neglecting higher twist contributions, the moments of the structure functions have the form

$$g_{j,n} := \int_0^1 \mathrm{d}x\, x^n g_j(x, Q^2) = A_{j,n} C_{j,n}(Q^2)\,, \tag{8}$$

where $A_{j,n}$ are the matrix elements of the leading twist nonsinglet operators and $C_{j,n}(Q^2)$ the corresponding Wilson coefficients. The Wilson coefficients can be calculated using their connection with the forward Compton scattering amplitude

$$\begin{aligned} t_{\mu\nu} = &-\frac{m(p_\mu S_\nu + S_\mu p_\nu)}{p\cdot q}\, 2\sum_n a_{3,n} C_{3,n}(Q^2)\omega^{n+1} \\ &+\frac{2m\, S\cdot q\, g_{\mu\nu}}{p\cdot q}\, 2\sum_n a_{5,n} C_{5,n}(Q^2)\omega^{n+1} + \dots\,, \end{aligned}\tag{9}$$

where $t_{\mu\nu}$ and $a_{j,n}$ refer to quark states instead of nucleon states. Adopting a normalization where the non-vanishing Wilson coefficients take the form

$$C_{j,n}(Q^2) = 1 + O(g^2) \tag{10}$$

the matrix elements of the leading twist operators are

$$\begin{aligned} a_{3,n} &= 2\,VA\,, \qquad n = 0, 2, 4\dots \\ a_{5,n} &= VA\,, \qquad n = 1, 3, 5\dots \end{aligned}\tag{11}$$

with the vector coupling constant V and the axial coupling constant A. The Borel transformed Wilson coefficients are now obtained by the calculation of $B[t_{\mu\nu}]$ and comparing the result expanded in ω with (9). In the calculations we have to handle the matrix γ_5 in $d \neq 4$ dimensions. We use the t'Hooft-Veltman scheme $\gamma_5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$, $\{\gamma_5, \gamma^\mu\} = 0$ for $\mu = 0, 1, 2, 3$ and $[\gamma_5, \gamma^\mu] = 0$ otherwise [22]. We get

$$\begin{aligned} &B[C_{3,n}](s) = \\ &C_F\left(\frac{\mu^2}{Q^2}\right)^{2-\frac{d}{2}}\left(\frac{\mu^2\mathrm{e}^{-c}}{Q^2}\right)^s \frac{\Gamma(\frac{d}{2})\Gamma(\frac{d}{2}-2-s)}{(4\pi)^{d/2}\Gamma(s+1)\Gamma(d-1-s)} \\ &\times\Bigg\{(d-2)\frac{\Gamma(s+n+3-\frac{d}{2})}{n!} + \left(s+2-\frac{d}{2}\right) \\ &\qquad\times\Big((d-4)-2\Big)\frac{\Gamma(s+n+2)\Gamma(s+n+4-\frac{d}{2})}{n!\Gamma(s+n+4)} \\ &+\frac{d}{2}(6-d)\frac{n\Gamma(s+n+2)\Gamma(s+n+3-\frac{d}{2})}{n!\Gamma(s+n+4)} \\ &+\frac{d}{4}\Big(4(d-4)^2 - 4(d-4)(d-2) + (d-2)^2\Big) \\ &\qquad\times\frac{\Gamma(s+n+2)\Gamma(s+n+3-\frac{d}{2})}{n!\Gamma(s+n+4)} \\ &+(d-2-s)(d-4)\frac{n\Gamma(s+n+1)\Gamma(s+n+3-\frac{d}{2})}{n!\Gamma(s+n+3)} \\ &+4\left(s+2-\frac{d}{2}\right)\sum_{k=0}^{n}\frac{\Gamma(s+k+3-\frac{d}{2})}{k!(s+k+1)} \\ &-2d\sum_{k=0}^{n}\frac{\Gamma(s+k+3-\frac{d}{2})}{k!(s+k+2)} \\ &+\left((2s+4-d)\frac{d-4}{2}-d\right)\sum_{k=0}^{n}\frac{k\Gamma(s+k+2-\frac{d}{2})}{k!(s+k+1)} \\ &+d\sum_{k=0}^{n}\frac{k\Gamma(s+k+2-\frac{d}{2})}{k!(s+k+2)} \\ &+\left(\frac{d}{2}-2-s\right)\Big(2(d-4)^2 - 2(d-4)(d-2) + (d-2)^2\Big) \\ &\qquad\times\sum_{k=0}^{n}\frac{\Gamma(s+k+2-\frac{d}{2})}{k!(s+k+1)} \\ &+(d-4-2s)(d-2-s)\frac{d-4}{d-2}\sum_{k=0}^{n}\frac{k\Gamma(s+k+2-\frac{d}{2})}{k!(s+k)}\Bigg\} \end{aligned}\tag{12}$$

for $n = 0, 2, 4\dots$ and

$$
\begin{aligned}
&B[C_{5,n}](s) = \\
&C_F \left(\frac{\mu^2}{Q^2}\right)^{2-\frac{d}{2}} \left(\frac{\mu^2 \mathrm{e}^{-c}}{Q^2}\right)^s \frac{\Gamma(\frac{d}{2})\Gamma(\frac{d}{2}-2-s)}{(4\pi)^{d/2}\Gamma(s+1)\Gamma(d-1-s)} \\
&\times\Bigg\{(d-2)\frac{\Gamma(s+n+3-\frac{d}{2})}{n!} \\
&+\left(s+2-\frac{d}{2}\right)\Big(2-(d-4)\Big) \\
&\qquad\times\frac{\Gamma(s+n+1)\Gamma(s+n+4-\frac{d}{2})}{n!\Gamma(s+n+3)} \\
&+\left(s+2-\frac{d}{2}\right)\Big((d-4)-2\Big) \\
&\qquad\times\frac{n\Gamma(s+n+1)\Gamma(s+n+3-\frac{d}{2})}{n!\Gamma(s+n+3)} \\
&+\bigg(2(d-2-s)(d-4)^2+\frac{1}{2}(6s+12-5d)(d-4)(d-2) \\
&\qquad+\frac{1}{4}(3d-8-4s)(d-2)^2\bigg) \\
&\qquad\times\frac{\Gamma(s+n+1)\Gamma(s+n+3-\frac{d}{2})}{n!\Gamma(s+n+3)} \\
&+4\left(s+2-\frac{d}{2}\right)\sum_{k=0}^{n}\frac{\Gamma(s+k+3-\frac{d}{2})}{k!(s+k+1)} \\
&-2d\sum_{k=0}^{n}\frac{k\Gamma(s+k+2-\frac{d}{2})}{k!(s+k+1)} \\
&+\left(\frac{d}{2}-2-s\right)\Big(2(d-4)^2-2(d-4)(d-2)+(d-2)^2\Big) \\
&\qquad\times\sum_{k=0}^{n}\frac{\Gamma(s+k+2-\frac{d}{2})}{k!(s+k+1)}\Bigg\}
\end{aligned}
\tag{13}
$$

for $n = 1, 3, 5 \dots$.

Since the NNA-approximation is exact in one loop order we get the next-to-leading-order result from (12) and (13) by taking $s = 0$ according to (4). An expansion in $\epsilon = 2 - \frac{d}{2}$ leads to

$$
\begin{aligned}
C_{3,n} = 1 + C_F\frac{g^2}{(4\pi)^2}\Bigg\{&\left(\frac{1}{\epsilon}-\gamma+\ln\frac{4\pi Q^2}{\mu^2}\right) \\
&\times\left(-4+\frac{4}{n+1}+\frac{4}{n+2}-\frac{4}{n+3}+4S_n\right) \\
&-\frac{3}{2}+\frac{9}{n+1}-\frac{6}{n+3}+\left(3+\frac{2}{n+2}-\frac{4}{n+3}\right)S_n \\
&+4\sum_{k=1}^{n}\frac{1}{k+2}S_k+2\sum_{k=1}^{n}\frac{1}{(k+1)(k+2)}S_{k-1}\Bigg\},
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
C_{5,n} = 1 + C_F\frac{g^2}{(4\pi)^2}\Bigg\{&\left(\frac{1}{\epsilon}-\gamma+\ln\frac{4\pi Q^2}{\mu^2}\right) \\
&\times\left(-3+\frac{2}{n+1}+\frac{2}{n+2}+4S_n\right) \\
&-1-\frac{4}{n+1}+\frac{6}{n+2}+\left(3-\frac{2}{n+1}+\frac{2}{n+2}\right)S_n \\
&+8\sum_{k=1}^{n}\frac{1}{k+1}S_{k-1}\Bigg\},
\end{aligned}
\tag{15}
$$

where S_n is defined by $S_n := \sum_{k=1}^{n}\frac{1}{k}$. From the last two equations we read off the renormalization constants for the corresponding composite operators (defined by $\mathcal{O}_r = Z^{-1}\mathcal{O}_0$) in the $\overline{\mathrm{MS}}$-scheme [23].

$$
\begin{aligned}
Z_{g_3,n} = 1 + C_F\frac{g^2}{(4\pi)^2}&\left(\frac{1}{\epsilon}-\gamma+\ln 4\pi\right) \\
&\times\left(-4+\frac{4}{n+1}+\frac{4}{n+2}-\frac{4}{n+3}+4S_n\right),
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
Z_{g_5,n} = 1 + C_F\frac{g^2}{(4\pi)^2}&\left(\frac{1}{\epsilon}-\gamma+\ln 4\pi\right) \\
&\times\left(-3+\frac{2}{n+1}+\frac{2}{n+2}+4S_n\right).
\end{aligned}
\tag{17}
$$

Finally we get for the anomalous dimensions $\gamma := \frac{\mu}{Z}\frac{\partial Z}{\partial\mu}\Big|_{g_0}$ (see e. g. [24]) in one loop order

$$
\begin{aligned}
\gamma_{g_3,n} = C_F\frac{g^2}{(4\pi)^2}& \\
&\times\left(8-\frac{8}{n+1}-\frac{8}{n+2}+\frac{8}{n+3}-8\sum_{k=1}^{n}\frac{1}{k}\right),
\end{aligned}
\tag{18}
$$

$$
\gamma_{g_5,n} = C_F\frac{g^2}{(4\pi)^2}\left(6-\frac{4}{n+1}-\frac{4}{n+2}-8\sum_{k=1}^{n}\frac{1}{k}\right).
\tag{19}
$$

To our knowledge these anomalous dimensions have not been calculated before. Higher order results could be obtained in the—no longer exact—NNA-approximation as well using (4).

To investigate the renormalons we can set $d = 4$. From (12) and (13) we get

$$
\begin{aligned}
&B[C_{3,n}](s) = \\
&C_F\left(\frac{\mu^2 e^{-c}}{Q^2}\right)^s\frac{1}{(4\pi)^2\Gamma(s+1)(2-s)s}\cdot\frac{1}{s-1} \\
&\times\Bigg\{2\frac{\Gamma(s+n+1)}{n!}-2s\frac{\Big(\Gamma(s+n+2)\Big)^2}{n!\Gamma(s+n+4)} \\
&+4\frac{n\Gamma(s+n+2)\Gamma(s+n+1)}{n!\Gamma(s+n+4)} \\
&+4\frac{\Gamma(s+n+2)\Gamma(s+n+1)}{n!\Gamma(s+n+4)} \\
&+2\sum_{k=0}^{n}\bigg[2s\frac{\Gamma(s+k+1)}{k!(s+k+1)}-4\frac{\Gamma(s+k+1)}{k!(s+k+2)} \\
&\quad-2\frac{k\Gamma(s+k)}{k!(s+k+1)}+2\frac{k\Gamma(s+k)}{k!(s+k+2)}-2s\frac{\Gamma(s+k)}{k!(s+k+1)}\bigg]\Bigg\},
\end{aligned}
\tag{20}
$$

$$B[C_{5,n}](s) = C_F\left(\frac{\mu^2 e^{-c}}{Q^2}\right)^s \frac{1}{(4\pi)^2\Gamma(s+1)(2-s)s}\cdot\frac{1}{s-1} \times\Bigg\{2\frac{\Gamma(s+n+1)}{n!}+2s\frac{\Gamma(s+n+1)}{n!(s+n+2)} -2s\frac{n\Big(\Gamma(s+n+1)\Big)^2}{n!\Gamma(s+n+3)}+4(1-s)\frac{\Big(\Gamma(s+n+1)\Big)^2}{n!\Gamma(s+n+3)} +4\sum_{k=0}^{n}\left[s\frac{\Gamma(s+k+1)}{k!(s+k+1)}-2\frac{k\Gamma(s+k)}{k!(s+k+1)}-s\frac{\Gamma(s+k)}{k!(s+k+1)}\right]\Bigg\}. \tag{21}$$

The pole at $s = 0$ corresponds to the usual $1/\epsilon$ pole in dimensional regularization. In both cases we find contributions from the two IR-renormalons at $s = 1$ and $s = 2$. The corrections corresponding to these renormalons are suppressed by factors $\frac{1}{Q^2}$ or $(\frac{1}{Q^2})^2$ respectively. We adopt the hypothesis that they dominate these power corrections [2–4]. For the dominant pole at $s = 1$ and taking $\mu^2 = Q^2$ we find for the residues

$$\mathrm{Res}\Big(B[C_{3,n}](s)\Big)\Big|_{s=1} = -\frac{C_F e^{-c}}{(4\pi)^2}\Bigg\{2n+6-\frac{4}{n+1}-\frac{4}{n+2} -\frac{16}{n+3}+\frac{24}{n+4}-4\sum_{k=1}^{n}\frac{1}{k}\Bigg\}, \tag{22}$$

$$\mathrm{Res}\Big(B[C_{5,n}](s)\Big)\Big|_{s=1} = -\frac{C_F e^{-c}}{(4\pi)^2}\Bigg\{2n+10-\frac{8}{n+1} -\frac{4}{n+2}-\frac{8}{n+3}-8\sum_{k=1}^{n}\frac{1}{k}\Bigg\}, \tag{23}$$

which are connected with the renormalon contributions according to (5) by

$$\Delta C_{j,n}(Q^2) = \pm\left(\frac{\Lambda^2}{Q^2}\right)\frac{1}{\beta_0}\mathrm{Res}\Big(B[C_{j,n}](s)\Big)\Big|_{s=1} +O\left(\frac{g^2}{Q^2},\frac{1}{Q^4}\right). \tag{24}$$

According to the heuristic theoretical framework (24) is supposed to give only the order of magnitude of power corrections. However despite the fact that the renormalon method has been discussed controversially [25] it has been checked experimentally in those cases for which data are available that it leads to reasonable estimates for the x-dependence of the power corrections [26]. The normalization turned also out to be reasonable, but this might be accidental. In the framework of the equivalent dispersive approach [9] the assumption was made that the prefactor defining the normalization has an universal character [27]. We should also note that ambiguities may arise in the x-dependence of power corrections depending on the choice of the argument of the running coupling. The discrepancy observed in [28] has not yet been understood completely.

According to (8) and (24) we get for the complete structure functions

$$g_{j,n}(Q^2) = A_{j,n}\Big[\sum_{k=0}^{N_0} C_{j,n}^k(Q^2)(g^2)^k + \Delta C_{j,n}(Q^2)\Big] \tag{25}$$

with the perturbative expansion of the Wilson coefficients $C_{j,n}=\sum_k C_{j,n}^k(g^2)^k$ and for the renormalon corrections of the same structure functions

$$\Delta g_{j,n}(Q^2) = A_{j,n}\Delta C_{j,n}(Q^2)\,. \tag{26}$$

The unknown matrix elements $A_{j,n}$ are eliminated taking the ratio

$$\begin{aligned}\frac{\Delta g_{j,n}(Q^2)}{g_{j,n}(Q^2)} &= \frac{\Delta C_{j,n}(Q^2)}{\sum_{k=0}^{N_0} C_{j,n}^k(g^2)^k+\Delta C_{j,n}(Q^2)}\\ &= \frac{\pm\left(\frac{\Lambda^2}{Q^2}\right)\frac{1}{\beta_0}\mathrm{Res}\Big(B[C_{j,n}](s)\Big)\Big|_{s=1}+O\left(\frac{g^2}{Q^2},\frac{1}{Q^4}\right)}{1+\sum_{k=1}^{N_0} C_{j,n}(Q^2)^k(g^2)^k+\Delta C_{j,n}(Q^2)}\\ &= \left[\pm\left(\frac{\Lambda^2}{Q^2}\right)\frac{1}{\beta_0}\mathrm{Res}\Big(B[C_{j,n}](s)\Big)\Big|_{s=1}+O\left(\frac{g^2}{Q^2},\frac{1}{Q^4}\right)\right]\\ &\quad\times\left[1+O\left(g^2,\frac{1}{Q^2}\right)\right]\\ &= \pm\left(\frac{\Lambda^2}{Q^2}\right)\frac{1}{\beta_0}\mathrm{Res}\Big(B[C_{j,n}](s)\Big)\Big|_{s=1}+O\left(\frac{g^2}{Q^2},\frac{1}{Q^4}\right).\end{aligned} \tag{27}$$

So in leading order the corrections are given by

$$\Delta g_{j,n}(Q^2) = \pm\left(\frac{\Lambda^2}{Q^2}\right)\frac{1}{\beta_0}\mathrm{Res}\Big(B[C_{j,n}](s)\Big)\Big|_{s=1}\cdot g_{j,n}(Q^2)\,. \tag{28}$$

The determination of all moments is equivalent to expressing $\Delta g_j(x)$ as a convolution

$$\Delta g_j(x) = \pm\left(\frac{\Lambda^2}{Q^2}\right)\frac{1}{\beta_0}\int_x^1\frac{\mathrm{d}y}{y}C_j^{IR1}(y)g(x/y)\,, \tag{29}$$

where the functions $C_j^{IR1}(y)$ defined by

$$\mathrm{Res}\Big(B[C_{j,n}](s)\Big)\Big|_{s=1} = \int_0^1 \mathrm{d}y\, y^n C_j^{IR1}(y) \tag{30}$$

are obtained from (20) and (21):

$$C_3^{IR1}(y) = \frac{C_F\mathrm{e}^{-c}}{(4\pi)^2}\Bigg\{2\delta'(y-1)-6\delta(y-1)+4+4y +16y^2-24y^3-\frac{4}{(1-y)_+}\Bigg\}, \tag{31}$$

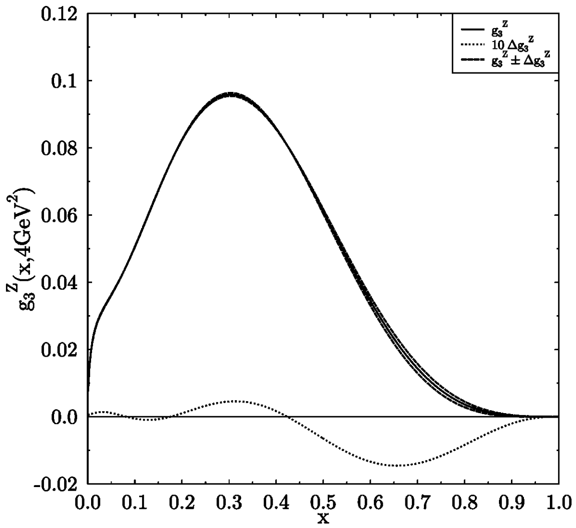

Fig. 1. The fit for g_3^Z (full line) and the corresponding renormalon contribution multiplied by a factor 10 (dotted line). The dashed lines show the size of the ambiguity for g_3^Z, i. e. $g_3^Z \pm \Delta g_3^Z$

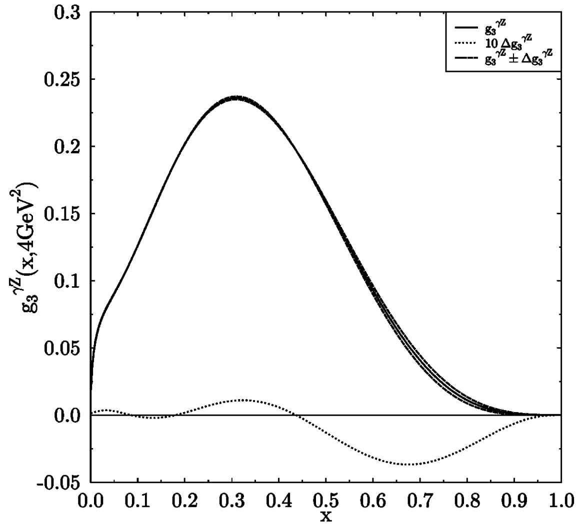

Fig. 3. The same as Fig. 1 for $g_3^{\gamma Z}$

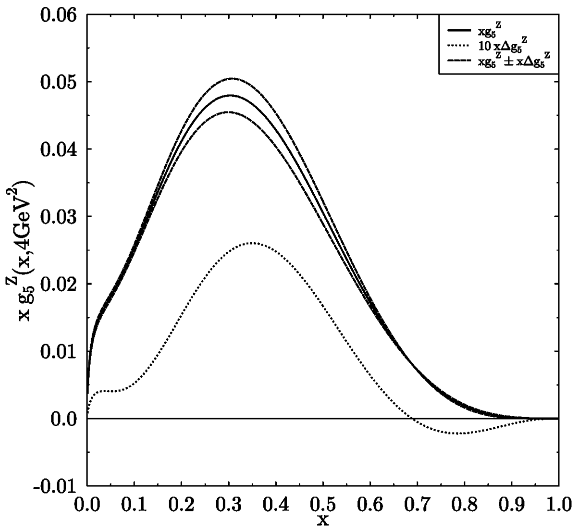

Fig. 2. The same as Fig. 1 for g_5^Z

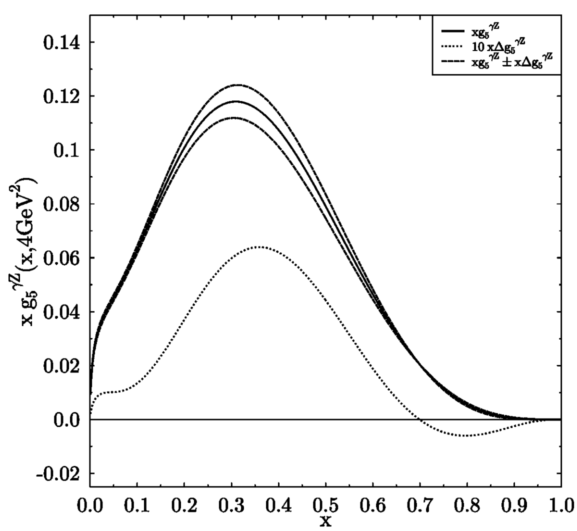

Fig. 4. The same as Fig. 1 for $g_5^{\gamma Z}$

$$C_5^{IR1}(y) = \frac{C_F \mathrm{e}^{-c}}{(4\pi)^2}\left\{2\delta'(y-1) - 10\delta(y-1) + 8 + 4y \right.$$
$$\left. + 8y^2 - \frac{8}{(1-y)_+}\right\}, \tag{32}$$

where $\frac{1}{(1-y)_+}$ is defined by $\int_0^1 \mathrm{d}y\, f(y) \frac{1}{(1-y)_+} = \int_0^1 \mathrm{d}y \frac{f(y)-f(1)}{1-y}$. We use the quark distributions given in [29] and the parton model expressions

$$g_3^Z = 2x \sum_q g_V^q g_A^q (\Delta q - \Delta\overline{q}) \,, \tag{33}$$

$$2xg_5^Z = g_3^Z \,, \tag{34}$$

$$g_5^{\gamma Z} = 2x \sum_q e^q g_A^q (\Delta q - \Delta\overline{q}) \,, \tag{35}$$

$$2xg_5^{\gamma Z} = g_5^{\gamma Z} \,. \tag{36}$$

We choose the momentum transfer to be $Q^2 = 4\,\mathrm{GeV}^2$. The integrals in (29) are evaluated numerically and the results are plotted in the figures 1 to 4.

We have thus completed our analysis of the renormalon ambiguities for all twist-2 structure functions. In all cases the size of these ambiguities turned out to be very small.

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